

CHANCHAL COLLEGE

Sub: Mathematics Honours

Class: B.Sc 1st year.

Assignment - 2017-18

(Test exam)

Full Marks - 20

Paper name: I-st paper.

Paper Code: I-A

Last date of submission: 02/02/2018

Choose the correct answer

$10 \times 2 = 20$

1. The general value of i^i is —

- a) $e^{-(4n+1)\pi}$ b) $e^{-(2n+1)\pi/2}$ c) $e^{-(4n+1)\pi/2}$ d) $e^{(4n+1)\pi/2}$.

2. The value of $1 \cdot 3 \cdot 5 \cdots (2n-1)$ is,

- a) $= n!$ b) $> n!$ c) $> n^n$ d) $< n^n$.

3. If α be a multiple root of the polynomial equation $f(x)=0$ of order r , then α be a multiple root of the polynomial equation $f'''(x)=0$ of order
a) $r-3$ b) r c) $r-2$ d) $r-1$.

4. If $\alpha + \beta + \gamma = 1$, $\alpha^2 + \beta^2 + \gamma^2 = 3$ and $\alpha^3 + \beta^3 + \gamma^3 = 7$ then,
 $\alpha^4 + \beta^4 + \gamma^4$ is,

- a) 7 b) 121 c) 11 d) 59.

5. Let A be a square matrix of order n and I be the identity of same order n , then $\det(\text{adj}(\text{adj } A))$ is equal to

- a) $(\det A)^{(n-1)^2}$ b) $(\det A)^{n^2}$ c) $(\det A)^{n-1}$ d) $(\det A)^{n-2}$

6. Let us consider a system of non-homogeneous equations $Ax=b$, where,

$$[A:b] = \begin{pmatrix} 1 & 1 & -2 & 1 & | & 1 \\ -1 & 2 & 3 & -1 & | & 0 \\ 0 & 3 & 1 & 0 & | & 1 \end{pmatrix}$$

Which of the following is true?

- a) The system has no solution.
- b) The system has infinite solutions.
- c) The system has unique solution.
- d) The system has finite solutions.

7. Consider the following subspace of \mathbb{R}^3 $W = \{(x,y,z) \in \mathbb{R}^3 : 2x+2y+z=0, 3x+3y-2z=0, x+y-3z=0\}$. The dimension of W is,

- a) 0
- b) 1
- c) 2
- d) 3.

8. Which one of the following is true?

- a) $A \cup \emptyset = \emptyset$
- b) $A \cap \emptyset = \emptyset$
- c) $A \cup A = \emptyset$
- d) $A \cap A = \emptyset$.

9. The permutation, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 4 & 5 & 3 & 2 \end{pmatrix}$ is,

- a) odd
- b) even
- c) even prime
- d) none of these.

10. The number of generators of the group $G = \{b, b^2, b^3, b^4, b^5, b^6 = e\}$ are,

- a) b^2, b^5
- b) b^3, b^4
- c) b, b^5
- d) b^3, b^5 .

CHANCHAL COLLEGE

Sub: Mathematics Honours

Class: B.Sc 1st year

Assignment - 2017-2018

Full Marks - 80

Last date of submission: 02/02/18

Paper code: I-B.

Group-A (Marks 30)

$$5 \times 6 = 30$$

Answer any six questions:-

- If $\tan(\alpha+i\beta) = \tan\theta + i\sec\theta$; where α, β, θ are real and $0 < \theta < \pi$
show that, $e^{i\beta} = \cot\theta/2$ and $\alpha = n\pi + \frac{\pi}{4} + \frac{\theta}{2}$; $n \in \mathbb{Z}$ (any integer)
- If $\alpha = \cos \frac{2n\pi}{n} + i\sin \frac{2n\pi}{n}$ and if n and p be prime to n ,
prove that, $1 + \alpha^p + \alpha^{2p} + \dots + \alpha^{(n-1)p} = 0$.
- If $\alpha, \beta, \gamma, \delta$ be the roots of the equation, $x^4 + 3x^2 + x + 1 = 0$.
Find the equation whose roots are, $2\alpha + \frac{1}{\alpha}, 2\beta + \frac{1}{\beta}, 2\gamma + \frac{1}{\gamma}, 2\delta + \frac{1}{\delta}$.
- Find the special roots of $x^{15}-1=0$. Deduce that, $2\cos \frac{2\pi}{15}, 2\cos \frac{4\pi}{15}, 2\cos \frac{8\pi}{15}, 2\cos \frac{16\pi}{15}$ are the roots of equation, $x^4 - x^3 - 4x^2 + 4x - 1 = 0$
- If $a_1, a_2, a_3, \dots, a_n$ be n positive rational numbers and
 $S = a_1 + a_2 + \dots + a_n$ then prove that,

$$\left(\frac{S}{a_1} - 1\right)^{a_1} \left(\frac{S}{a_2} - 1\right)^{a_2} \dots \left(\frac{S}{a_n} - 1\right)^{a_n} \leq (n-1)^S$$
- If n be positive integer, prove that,

$$\frac{1}{\sqrt{4n+1}} < \frac{3 \cdot 7 \cdot 11 \dots (4n-1)}{5 \cdot 9 \cdot 13 \dots (4n+1)} < \sqrt{\frac{3}{4n+3}}$$
- If a, b are integers, not both zero, then prove that, there exists integers u and v such that, $\gcd(a, b) = au+bv$.

8. Solve by Cardan's method, $x^3 - 27x - 54 = 0$.

Group-B
(Marks - 25)

Answer any five questions:-

$5 \times 5 = 25$

9. R is the relation defined on the set of integers \mathbb{Z} such that $R = \{(a, b) : a, b \in \mathbb{Z}; a-b = 5n; n \in \mathbb{Z}\}$. Show that, R is an equivalence relation. If $R' = \{(a, b) : a, b \in \mathbb{Z}, a-b=3n, n \in \mathbb{Z}\}$, show that, the relation, RUR' is symmetric but not transitive [3+2]
10. Prove that, every field is an integral domain. Is the converse true? Justify your answer with a suitable example.
11. State and prove Lagrange's theorem for a finite group [2+4]
12. Prove that, a finite semi-group in which both the cancellative laws hold is a group.
13. Prove that, a non-trivial finite ring having no divisors of zero is a ring with unity. Define an Integral domain [4+1]
14. Prove that, the set $S = \{a + ib\sqrt{5} : a, b \in \mathbb{R}\}$ is a sub-field of \mathbb{C} .
Prove that, the ring of matrices $\left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in \mathbb{R} \right\}$ is a field.
15. Show that, an element m of the ring $\{Z_n, \oplus, \otimes\}$ will be unit if and only if m, n are relatively prime.
16. Prove that, the characteristic of an Integral domain is either zero or a prime number.

7. Let 'a' be fixed element in a ring R and let $C(a) = \{x \in R \mid ax = xa\}$. Prove that $C(a)$ is a subring of R.

$(D, +, \cdot)$ is an integral domain and $a, b \in D$. If $a^5 = b^5$ and $a^8 = b^8$. Prove that, $a = b$.

Group - C
(Marks - 25)

$5 \times 5 = 25$

16. $\begin{vmatrix} x & a & b & c \\ -a & x & d & e \\ -b & -d & x & f \\ -c & -e & -f & x \end{vmatrix} = x^4 (a^2 + b^2 + c^2 + d^2 + e^2 + f^2)x^2 + (af - be + cd)^2$

Hence prove that, the skew-symmetric determinant of even order 4 is a perfect square.

17. Prove that, a real symmetric matrix A is positive definite if and only if $A = BB^t$ for some non-singular matrix B. Show that, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 11 \end{bmatrix}$ is positive definite and find the matrix B such that, $A = BB^t$

18. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then use Cayley-Hamilton's theorem to show that,

$$2A^5 - 3A^4 + A^2 - 4I_2 = 138A - 403I_2.$$

19. Find the values of λ and μ for which the following equations,

$$x+y+z=6$$

$$x+2y+3z=10$$

$$x+2y+\lambda z=\mu$$

have -

a) no solution b) an infinite number of solutions.

20. Show that, every linearly independent subset of a finite dimensional vector space $V(F)$ is either a basis or can be extended to a basis.

21. Show that, the set $S = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$ is a basis of vector space $\mathbb{R}_{2 \times 2}^+$.

22. Apply Gram-Schmidt process to obtain an orthonormal basis of the subspace of Euclidean space \mathbb{R}^4 with standard inner product spanned by the vectors, $(1, 1, 0, 1), (1, -2, 0, 0), (1, 0, -1, 2)$