

CHANCHAL COLLEGE

ASSIGNMENT - 2017-18.

Mathematics HONS / III-Year / VI - A / 20.

Paper Name: VI.

Full Marks: 20.

Paper part:

Last date of submission: 25.01.18

Assignment will be submitted in A4 size paper with front page bearing Name, dept, Roll No. etc. to Dr. Animesh Kundu. Late submission will not be entertained.

1. Answer any twenty questions. $(20 \times 1) = 20$. [Choose the correct answer].

(i) A bag contains 5 brown and 4 white balls. A man pulls out two balls from the bag. The prob. that they are of the same colour is

- (a) $\frac{1}{5}$ (b) $\frac{3}{7}$ (c) $\frac{4}{9}$ (d) None of

(ii) The p.d.f of a random variable X is $f(x) = k(2x-1)$, $0 \leq x \leq 2$. Then the value of constant k is

- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$.

(iii) For a random variable X , $E\{(X-2)^2\} = 6$, $E\{(X-1)^2\} = 10$; then σ_x is

- (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 0 (d) 2.

(iv) The mean and S.D of a Binomial distribution are 4 and $\sqrt{\frac{8}{3}}$ respectively. Then the values of n & P are

- (a) 11, $\frac{3}{4}$ (b) 12, $\frac{2}{7}$ (c) 12, $\frac{1}{3}$ (d) 11, $\frac{4}{3}$.

(v) If X and Y are two independent χ^2 -variate with degrees of freedom (d.o.f) 3 and 4 then their sum $X+Y$ is

- (a) χ^2 -variables (d.o.f 12) (b) Normal variables (c) χ^2 -(d.o.f 7) (d) χ^2 -(d.o.f 1)

(vi) If X, Y, Z are three independent S.N.D (0,1) then $X^2 + Y^2 + Z^2$ is a

- (a) ~~Standard normal variables~~ (b) χ^2 (d.o.f 3) (c) t -(d.o.f 3) (d) ~~F distribution with k & n~~

(vii) If X is a $(2000, \frac{1}{2})$ binomial variates $P(|X - 1000| \geq 100) \leq a$
 then a is
 (a) $\frac{1}{20}$ (b) $\frac{19}{20}$ (c) $\frac{9}{20}$ (d) None of these.

(viii) A statistic t is called an unbiased estimator of a parameter θ when
 (a) $E(t) = \theta$ (b) $E(t^r) = \theta$ (c) $[E(t)]^r = [E(\theta)]^r$
 (d) $E(t^r) = \theta^r$.

(ix) The confidence intervals for the parameter μ of a Normal population with parameter μ, σ (when σ is known) is

(a) $(\bar{x} + \frac{\sigma}{\sqrt{n}} z_c, \bar{x} - \frac{\sigma}{\sqrt{n}} z_c)$ (b) $(\bar{x} - \frac{\sigma}{\sqrt{n}} z_c, \bar{x} + \frac{\sigma}{\sqrt{n}} z_c)$

(c) $(\bar{x} - \frac{\sigma}{n} z_c, \bar{x} + \frac{\sigma}{n} z_c)$ (d) None

(x) If $H_0 (\theta = 2)$ is Null hypothesis then the prob. of rejection of H_0 though $\theta = 2$ is true is prob. of

(a) Type I - error (ii) Type - II error (iii) $H_0 (\theta \neq 2)$; (d) None.

(xi) If $\{x_1, x_2, \dots, x_n\}$ is a sample drawn from a population which has poisson distribution with parameter m , then if

$\bar{x} = (x_1 + x_2 + \dots + x_n) / n$, then m.o.e of m is

(a) \bar{x} (b) \bar{x}/n (c) $2\bar{x}$ (d) $n\bar{x}$.

(xii) If μ be the mean of a Normal population and the value of a sample mean $\bar{x} = 3$ where $P(-2.58 < \frac{\bar{x} - \mu}{0.28} < 2.58) = 0.99$ then the 99% confidence interval of μ is

(a) $(0, 3.72)$, (b) $(-0.28, 2.28)$, (c) $(2.28, 3.72)$

(d) $(1.028, 4.72)$

(iii) The value of $\left(\frac{\Delta^r}{E}\right) x^r$ is (a) 2 (b) 3 (c) 4 (d) 6.

(xiv) The missing term in the following table:

x:	0	1	2	3	4
y:	1	2	4	?	16

is (a) 8 (b) 7 (c) 8.25 (d) 8.45.

(xv) The 3rd ~~year~~ order divided difference of the function $f(x) = \frac{1}{x}$ with arguments a, b, c, d is

(a) $\frac{1}{abcd}$ (b) $-\frac{1}{abd}$ (c) $-\frac{1}{abcd}$ (d) $-\frac{1}{abe}$.

(xvi) The iteration scheme $x_{n+1} = \frac{x_n}{2} \left(1 + \frac{a}{x_n^2}\right)$ converges to \sqrt{a} .

The convergence is

(a) linear (b) quadratic (c) Cubic (d) None.

(xvii) The 4th order Runge-Kutta Method is of

(a) $O(h^3)$ (b) $O(h^2)$ (c) $O(h^4)$ (d) $O(h^5)$

(xviii) The condition of convergence of Newton-Raphson Method is

(a) $f'(x) \neq 0$ (b) $|f'(x)| < 1$ (c) $\{f'(x)\}^2 > |f(x)f''(x)|$
(d) $\{f''(x)\}^2 > |f(x)f'(x)|$.

(xix) In LU factorization Method, a Matrix A can be factorized in $A = LU$ where L is a

(a) upper triangular (b) Lower triangular (c) diagonal
(d) None matrix.

(xx) The degree of Precision in Simpson's $\frac{1}{3}$ Rule is

(a) 1 (b) 2 (c) 3 (d) d.

(xxi) In Simpson's $\frac{1}{3}$ Rule for finding $\int_a^b f(x) dx$, $f(x)$ is approximated by

(a) line segment (b) Parabola (c) Circular sector (d) ellipse.

(xxii) Lagrange's interpolation formula can be used for the arguments

(a) Equispaced (b) Unequally spaced (c) both (a) & (b).
(d) None of these.

(xxiii) The significant digit of 0.0001234 is (a) 7 (b) 4 (c) 8 (d) 6.

Answer any 3 questions: $3 \times 10 = 30$, G15-A.

2. (a) A random point (X, Y) is uniformly distributed over a circular region $x^2 + y^2 < a^2$. Find the marginal distribution of X and Y and the conditional distribution of Y assuming $X=x$, where $|x| < a$. [6]

or

The joint density function of the random variables X, Y is given by

$$f_{X,Y}(x,y) = x+y; \quad 0 < x < 1, \quad 0 < y < 1$$

= 0 elsewhere
find the distribution of $X+Y$.

- (b) Two people agree to meet at a definite place between 12 and 1 O'clock with the understanding that each will wait 20 min. for the other. What is the prob. that they will meet. [4]

3. (a) If X be a normal (m, σ) variates, then prove that

$$\mu_{2r+2} = \sigma^2 \mu_{2r} + \sigma^3 \frac{d\mu_{2r}}{d\sigma}, \quad \text{Hence find the Co-efficient of Kurtosis } \beta_2 \text{ of this distribution. [5]}$$

or

- (b) The Joint Prob. density function of a Random Variables X and Y is $\frac{1}{2} x^3 e^{-x(y+1)}$, $(0 < x < \infty, 0 < y < \infty)$, determine the correlation coefficient of Y on X . [5].

or

If X_1, X_2, \dots, X_n are mutually independent normal variables with parameters $(m_1, \sigma_1), (m_2, \sigma_2), \dots, (m_n, \sigma_n)$ resp. and a_1, a_2, \dots, a_n be any real constants then ~~or~~ prove that $a_1 X_1 + a_2 X_2 + \dots + a_n X_n$ follows normal distribution with parameters $(a_1 m_1 + a_2 m_2 + \dots + a_n m_n, \sqrt{a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + \dots + a_n^2 \sigma_n^2})$.

4. If $\{X_i\}_i$ be a sequence of independent random variables such that for each i , $E(X_i) = m$, $\text{Var}(X_i) = \sigma_i^2$, $\sigma_i^2 < \infty$, use Tchebycheff's inequality to prove that,

$$\sum_{i=1}^n \frac{X_i}{n} - \sum_{i=1}^n \frac{m_i}{n} \xrightarrow{\text{in P}} 0 \text{ as } n \rightarrow \infty. \quad [5]$$

- (b) If x_1, x_2, \dots, x_n is a random sample from an infinite population with variance σ^2 and \bar{x} is the sample mean, show that $\sum_{i=1}^n (x_i - \bar{x})^2 / n$ is a biased estimator of σ^2 , but the bias become negligible for large n . Give an unbiased estimator of σ^2 . [5]

- (5) (a) A population is defined by $f(x, \theta) = \frac{x^{p-1} e^{-x/\theta}}{\theta^p \Gamma(p)}$, $0 < x < \infty$, where p is known, $p > 0$, find the m.l.e of $\theta > 0$, drawing a sample x_1, x_2, \dots, x_n from the population. [5]

- (5) The random variables X denoting the amount of consumption of a product follows the distribution

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 < x < \infty, \quad \theta > 0$$

The hypothesis $H_0: \theta = 5$ is rejected in favour of $H_1: \theta = 10$ if 15 units or more, chosen randomly be assumed.

Obtain the size of Type-I and Type-II errors. [5]

- 6 (i) Find by the method of likelihood ratio testing, a test of $H_0: \sigma = \sigma_0$ for a Normal Population (m, σ) assuming m is known. [6]

- (b) Obtain 99% confidence interval of the population standard deviation (σ) on the basis of the data $\sum_{i=1}^{10} x_i = 620$, $\sum_{i=1}^{10} x_i^2 = 39016$ [4]

Answers any four questions: $4 \times 10 = 40$.

Gr-B

8.9 Explain the Gauss-elimination method to solve a systems of n equations in n unknowns. comments on the convergence of the Gauss-Seidel iteration method.

b) Establish Newton-Cotes quadrature formula for Numerical integration of $f(x)$ in $[a, b]$ for equal arguments.

What do you mean by closed and open type quadrature formula.

determine a, b, c such that the formula $\int_a^b f(x) dx = h[a f(a) + b f(b) + c f(c)]$ is exact for polynomials of as higher order as possible.

9. (a) If $y = f(x)$ be known for x_0, x_1, \dots, x_n arguments such that $x_i = x_0 + ih$ ($h > 0; i = 0, 1, 2, \dots, n$) then prove that $f[x_0, x_1, \dots, x_n] = \frac{h^n}{n!} \Delta^n y_0$.

(b) define k -th order difference of a function $f(x)$. Prove that $\Delta^k f(x) = \sum_{i=0}^k (-1)^i \binom{k}{i} f(x + (k-i)h)$.

10. (a) Deduce Modified Euler's Method for solving the following ODE,

$$\frac{dy}{dx} = f(x, y); \quad y(x_0) = y_0. \text{ Explain geometrically.}$$

(b) Using Runge-Kutta Method of 4th order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$. Compare it to original solution.

11. (a) Write a FORTRAN Programme to compute factorial $n!$.
 (b) Write a FORTRAN Programme to evaluate sum of the infinite series: $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots - \infty$ with error $< 10^{-6}$ (for some x).

12. (a) Write a FORTRAN Programme to find the roots of the quadratic equation $ax^2 + bx + c = 0$ (a, b, c are real $a \neq 0$).
 (b) Write a FORTRAN Programme whether the triangle is equilateral, then compute the area and perimeter of the equilateral triangle.

Write a FORTRAN Programme to find whether a given square matrix is symmetric.

13. a) Write a programme in C/FORTRAN to print K

$$\text{where } K = \sum_{I=1}^m \sum_{J=1}^n I * J$$

b) Write a C/FORTRAN Programme to read five numbers and ~~write a~~ ~~for~~ evaluate the mean and variance of these numbers.

or

Write a FORTRAN/C Programme which finds all prime numbers between 1000 and 9999.