

CHAN CHAL COLLEGE

Sub: Mathematics Honours

Class: B.Sc 1st year (2nd paper)

Assignment - 2017-2018

Subject code: I-A

Full marks - 20

Last date of submission -  
6.02.18

$$10 \times 2 = 20$$

1.  $\lim_{x \rightarrow 3} \frac{(x-3)}{|x-3|}$  is
- a) 0      b) 1      c) -1      d) does not exist
2. Let  $f$  be defined on  $[-1, 3]$  that,
- $$f(x) = \begin{cases} x & ; x \in \mathbb{Q} \\ 2-x & ; x \in \mathbb{R} - \mathbb{Q} \end{cases}$$
- then  $f$  is continuous in this interval at
- a) one point    b) two point    c) all point    d) no point
3. The maximum value of  $(\frac{1}{x})^x$ ;  $x > 0$  is
- a)  $e$     b)  $e^{-e}$     c)  $\frac{1}{e}$     d)  $e^{1/e}$
4. Let  $f(x)$  and  $g(x)$  be differentiable on  $0 \leq x \leq 2$  such that  $f(0)=2$ ,  $f(2)=5$ ,  $g(0)=0$  and  $f'(x)=g'(x)$  for all  $x \in (0, 2)$ , then the value of  $g(2)$  is
- a) -1    b) 1    c) 2    d) 3.
5. The discontinuity of the function,  $f(x) = \begin{cases} \sin \frac{1}{x} & ; x \neq 0 \\ 1 & ; x = 0 \end{cases}$  at  $x=0$  is, ...

- CONFIDENTIAL
- a) infinite discontinuity    b) removable discontinuity
  - c) discontinuity of second kind    d) oscillatory discontinuity

6. In Taylor's theorem of  $f(x) = e^x$  in  $[a, a+h]$ , the remain term of Lagrange's form of remainder after  $n$  term for  $0 < \theta < 1$  is,

- a)  $R_n = \frac{h^n}{n!} e^{a+oh}$
- b)  $R_n = \frac{h^n}{n!} e^{oh}$
- c)  $R_n = \frac{h^n(1-\theta)}{(n-1)!} e^{a+oh}$
- d)  $R_n = \frac{h^n}{n!} e^{a+(1-\theta)h}$

7. Let  $f_n(x) = x^n$  for  $x \in [0, 1]$ . Then,

- a)  $\lim_{n \rightarrow \infty} f_n(x)$  exists for all  $x \in [0, 1]$
- b)  $\lim_{n \rightarrow \infty} f_n(x)$  define a continuous function on  $[0, 1]$
- c)  $\{f_n\}$  converges uniformly on  $[0, 1]$
- d)  $\lim_{n \rightarrow \infty} f_n'(x) = 0$  for all  $x \in [0, 1]$

8. For all real  $x$ , the series,  $\sum_{n=1}^{\infty} \frac{x}{n+n^2x^2}$

- a) converges pointwise but not uniformly
- b) converges uniformly
- c) is neither pointwise nor uniform convergent
- d) none of these

9.  $x^3 \log(y/x)$  is homogeneous function of  $x$  and  $y$  of degree

- a) 0    b) 1    c) 2    d) 3

10. ~~Then and now~~ ~~Ex~~

10. If  $\lim_{n \rightarrow 0} \frac{f(a+n) - f(a-h)}{n}$  is equal to

- a)  $f(a)$
- b)  $f'(a)$
- c)  $2f'(a)$
- d) 0

# CHANCHAL COLLEGE

Sub: Mathematics Honours.

Class: B.Sc 1st year (2nd paper)

Assignment - 2017 - 2018

Full marks - 80

Subject code: I-B

Last date of submission -

5.02.18

Group-A

(Marks - 35)

$5 \times 7 = 35$

Answer any seven questions:

- If a real valued function  $f(x)$  is continuous in a closed interval  $[a, b]$  then  $f(x)$  is bounded in  $[a, b]$  and attains its bounds in  $[a, b]$
- Test the series  $\sum_{n=1}^{\infty} (\sqrt{n^2+1} - n^2)$  is convergence or not.
- State and prove nested interval theorem on sequence of closed intervals.
- Show that every Cauchy sequence of real numbers is convergent.
- State and prove Taylor's theorem with Cauchy's form of remainder.
- If a continuous function  $f: \mathbb{R} \rightarrow \mathbb{R}$  satisfies the relation  $f(x+y) = f(x) \cdot f(y)$ , then prove that, either  $f(x)=0$  or else  $f(x) = e^{ax}$  for all  $x$ . ( $a$  is constant).
- Let a function  $f$  be continuous on the open interval  $(a, b)$ . Prove that,  $f$  is uniformly continuous on  $(a, b)$  if and only if  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow b^-} f(x)$  both exist finitely.
- State and prove first mean value theorem of differential calculus. Hence or otherwise prove,

$$0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1 ; x > 0$$

[3+2]

9. State and prove D'Alembert's ratio test for convergence of series of positive terms.

[1+4]

10. State and prove, Rolle's theorem. Prove that between any two real roots of  $e^x \sin x = 1$ , there exists at least one root of  $e^x \cos x + 1 = 0$ .

[1+3+1]

Group-B.  
(Marks: 20)

$5 \times 4 = 20$

Answer any four questions:-

11. If  $H(x, y)$  be a homogeneous function of two real variables  $x$  and  $y$  of degree  $n$  having continuous first order partial derivatives and  $u(x, y) = (x^2 + y^2)^{-\frac{n}{2}}$ . Show that,

$$\frac{\partial}{\partial x} \left( H \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( H \frac{\partial u}{\partial y} \right) = 0.$$

12. If  $x^x y^y z^z = k$  (constant), prove that, at the point  $(x, y, z)$ , where  $x=y=z$ ,

$$Z_{xy} = -\frac{1}{x \log(ex)}$$

13. If  $u = f(x, y)$  and  $x = r \cos \theta$  and  $y = r \sin \theta$  then prove that,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}$$

14. Let  $D \subset \mathbb{R}^2$  and  $f: D \rightarrow \mathbb{R}$ . Prove that,  $f$  is differentiable at a point  $(a, b)$  interior to  $D$  if  $f_x$  exists at the point  $(a, b)$  and  $f_y$  is continuous at  $(a, b)$ .

15. Let  $f(x, y) = (x^2 + y^2) \tan^{-1} \frac{y}{x}$  ;  $x \neq 0$   
 $= \frac{\pi y^2}{2}$  ;  $x = 0$

then show that,  $f_{xy}(0, 0) \neq f_{yx}(0, 0)$

16. State and prove Young's theorem

[1+4]

Group-C  
(Marks - 25)

$5 \times 5 = 25$

17. Find the pedal equation of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with respect to focus of it.
18. Find the area of the loop of the curve,  
 $x(x^2+y^2) = a(x^2-y^2) \Rightarrow a > 0$
19. Show that, the evolute of the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$  is  
 $(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$ .
20. Find all the asymptotes of the curve,  
 $3x^3 + 2x^2y - 7xy^2 + 2y^3 - 14xy + 7y^2 + 4x + 5y = 0$ .
21. If  $I_n = \int_0^{\pi} \left( \frac{\sin nx}{\sin x} \right)^2 dx$ , then show that,  $I_{n+1} = 2I_n - I_{n-1}$   
and hence show that,  $I_n = n\pi$ .
22. If m and n are positive integers. Prove that,  
 $\int_a^b (x-a)^m (x-b)^n dx = (b-a)^{m+n+1} \cdot \frac{m! n!}{(m+n+1)!}$