

Paper-III
(Group-A)

1. Answer any 5 M.C.Q. type questions.

$$5 \times 2 = 10$$

- (a) If the pair of straight lines $x^r - 2pxy - y^r = 0$ and $x^r - 2qxy - y^r = 0$ be such that each pair bisects the angle between the other pair, then
 (i) $p = q$ (ii) $p = -q$ (iii) $pq = 1$ (iv) $pq = -1$.

- (b) If the normal at the point $(at_1^r, 2at_1)$ on a parabola meets the parabola again at the point $(at_2^r, 2at_2)$, then

$$(i) t_2 = -t_1 - \frac{2}{t_1} \quad (ii) t_2 = -t_1 + \frac{2}{t_1} \quad (iii) t_2 = t_1 - \frac{2}{t_1} \quad (iv) t_2 = t_1 + \frac{2}{t_1}$$

- (c) The locus of the middle points of a chord of the circle $x^r + y^r = 4$ which subtends a right angle at the origin is

$$(i) x + y = 2 \quad (ii) x^r + y^r = 1 \quad (iii) x^r + y^r = 2 \quad (iv) x + y = 1$$

- (d) The plane $x + 2y - z = 4$ cuts the Sphere $x^r + y^r + z^r - x + 2z - 2 = 0$ in a circle of radius - (i) 3 (ii) 1 (iii) 2 (iv) $\sqrt{2}$.

- (e) The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, if (i) $k = 0$ or -1 (ii) $k = 1$ or -1 (iii) $k = 0$ or -3 (iv) $k = 3$ or -3

- (f) If the particle describes a curve $r = ae^\theta$ with constant angular velocity. Then the radial acceleration is
 (i) constant (ii) zero (iii) ae (iv) None of these

(Group-B)

2. Answer any two questions : $2 \times 4 = 8$

- (a) Prove that two points on the conic $\frac{1}{r} = 1 + e \cos\theta$, whose vectorial angles are α, β will be extremities of a diameter if $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} + \frac{1+e}{1-e} = 0$.

- (b) Prove that if one line of the pair of straight lines $ax^r + 2hxy + by^r = 0$ be perpendicular to one line of the pair of straight lines represented by $a'x^r + 2h'xy + b'y^r = 0$ then

$$(aa' - bb')^2 + 4(ah' + hb')(a'h + h'b) = 0.$$

- (c) Show that the locus of the point of intersection of tangents to the parabola $y^2 = 4ax$ at points whose ordinates are in the ratio $p^r : q^r$ is $y^2 = \left(\frac{p}{q^r} + \frac{q^r}{p^r} + 2\right)ax$.

(Group - c)

3. Answer any two questions: $2 \times 5 = 10$

- (a) Find the length of the shortest distance between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{3}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$. Find also the co-ordinates of the points where the line of the shortest distance meets the given lines.
- (b) The plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α . Show that the equation of the plane in the new position is $ax + by \pm 2\sqrt{a^2 + b^2} \tan \alpha = 0$.
- (c) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 4$, $z = 0$ and cut by the plane $x + 2y + 2z = 0$ into a circle of radius 3.

(Group - D)

4. Answer any two questions: $2 \times 6 = 12$

- (a) A body of mass $(m_1 + m_2)$ is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E . Show that, if after explosion the parts move in the same line as before, then their relative speed is $\sqrt{\frac{2E(m_1+m_2)}{m_1m_2}}$.
- (b) A particle is projected vertically upwards with a velocity v in a medium whose resistance is Kv^4 per unit mass, v being the velocity of the particle and K is a constant. Show that greatest height reached is $\frac{1}{2\sqrt{Kg}} \tan^{-1} \left\{ \sqrt{\frac{Kv^2}{g}} \right\}$

(c) A particle moves with a central acceleration $\lambda^2(8au^2 + a^4u^5)$, where $u = \frac{1}{r}$. The particle is projected with velocity 9λ from an apse at a distance $\frac{a}{3}$ from the origin. Find out the equation of the path of the particle.

Paper - IV
(Group - E)

5. Answer any five MCQ type questions: $5 \times 2 = 10$

- (a) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to
 (i) 0 (ii) -7 (iii) 7 (iv) 1

- (b) A unit vector is orthogonal to $5\vec{i} + 2\vec{j} + 6\vec{k}$ and is coplanar to $2\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} - \vec{j} + \vec{k}$, then the vector is
 (i) $\frac{3\vec{j} - \vec{k}}{\sqrt{29}}$ (ii) $\frac{2\vec{i} + 5\vec{j}}{\sqrt{29}}$ (iii) $\frac{6\vec{i} - 5\vec{k}}{\sqrt{61}}$ (iv) $\frac{2\vec{i} + 2\vec{j} - \vec{k}}{3}$

- (c) Which of the following equations is an exact differential equation?
 (i) $(x^2 + 1)dx - ny dy = 0$ (ii) $x dy + (3x - 2y) dx = 0$
 (iii) $2ny dx + (2 + x^2) dy = 0$ (iv) $x^2 y dy - y dx = 0$

- (d) Determine the differential equation of the family of circles with centre on the y -axis
 (i) $y''' - xy'' + y' = 0$ (ii) $y'' - nxyy'' + y' = 0$
 (iii) $xy'' - y'^3 - y' = 0$ (iv) $y'^3 + y''^2 + nxy = 0$

- (e) The set $X = \{x : |x| \leq 2\}$ is
 (i) convex set (ii) not convex set (iii) Both i & ii (iv) None of these.
 (f) In maximization problem, optimal solution occurring at corner point yields the (i) Mean values of Z (ii) Highest values of Z
 (iii) Lowest value of Z (iv) Mid values of Z .

- (g) Which of the following is a valid objective function for a LPP?
 (i) Max $5x_1 + 6x_2$ (ii) Min $4x_1 + 3x_2 + \frac{2}{3}Z$
 (iii) Max $5x_1 + 6x_2$ (iv) Min $\frac{x_1 + x_2}{x_3}$

(Group - F)

6. Answer any two questions: $2 \times 4 = 8$

- (a) If $\nabla \phi(2xyz^3, x^2z^3, 3x^2yz^2)$ and $\phi(1, -2, 2) = 4$, find the function ϕ .

(b) Using vector method show that $\sin(\alpha - \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$.

- (c) Show that a proper vector function $\vec{f}(t)$ has a constant length if and only if $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$.

(Group - G)

7. Answer any two questions: $2 \times 6 = 12$

- (a) Find the general and singular solutions of $16x^2 + 2p^2y - p^3x = 0$.

(b) Using Method of variation of parameter solve the equation

$$y'' - 3y' + 2y = \frac{e^x}{1+e^x}$$

- (c) Solve by the method of undetermined coefficients the equation

$$\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = 4x^2 + 6x + 3 \cos 2x.$$

(Group - H)

8. Answer any two questions: $2 \times 5 = 10$

- (a) Define convex set. Prove that a hyperplane is a convex set. (2+3)

(b) Use two phase method to solve the L.P.P.

$$\text{Maximize } Z = 2x_1 + 3x_2 + x_3$$

$$\text{Subject to } 3x_1 - 2x_2 - 3x_3 + 8 = 0$$

$$3x_1 - 4x_2 - 2x_3 + 7 = 0$$

$$x_1, x_2, x_3 \geq 0.$$

- (c) Find the optimal solution of the Transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
	5	8	7	14	
b_j					