

**CHANCHAL COLLEGE****ASSIGNMENT- 2021****MATHEMATICS (HONOURS)****Paper Code: IV-A**

Full Marks-20

Time: Thirty minuits

Answer all the questions. Each question carries 2 marks.

- Feasible region in the set of points which satisfy
  - The objective functions
  - Some the given constraints
  - All of the given constraints
  - None of these
- A set of values of decision variables which satisfies the linear constraints and non-negativity conditions of a L.P.P. is called its
  - Unbounded solution
  - Optimum solution
  - Feasible solution
  - None
- The maximum value of  $Z = 4x + 2y$  subject to the constraints  $2x + 3y \leq 18$ ,  $x + y \geq 10$ ,  $x, y \geq 0$  is
  - 36
  - 40
  - 30
  - None of these
- Maximize  $Z = 3x + 5y$ , subject to  $x + 4y \leq 24$ ,  $3x + y \leq 21$ ,  $x + y \leq 9$ ,  $x \geq 0$ ,  $y \geq 0$ 
  - 20 at (1, 0)
  - 30 at (0, 6)
  - 37 at (4, 5)
  - 33 at (6, 3)
- Integrating factor of  $\frac{dr}{d\theta} = 500\theta^n - \frac{r}{\theta}$ 
  - $\theta$
  - $2\theta$
  - $3\theta$
  - $4\theta$
- Integrating factor of  $y \frac{dx}{dy} = -2x + 10y^3$ 
  - $y$
  - $y+1$
  - $y+3$
  - none
- The DE  $(1+(y')^2)^3 = r^2 \left( \frac{d^2y}{dx^2} \right)$  represents
  - circle of radius "r"
  - sphere of radius "r"
  - ellipse
  - parabola
- The integrating factor of  $x \log x \frac{dx}{dy} + y = 2 \log x$ 
  - $\log x$
  - $\log 2x$
  - $\log 3x$
  - $\log 4x$
- The directional derivatives of  $1/r$  in the direction of  $\mathbf{r}$  is
  - $\frac{1}{r^3}$
  - $\frac{1}{r^2}$
  - $\frac{1}{r}$
  - None of these
- The value of t for which  $\mathbf{A} + t\mathbf{B}$  is perpendicular to  $\mathbf{C}$  where  $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{B} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and  $\mathbf{C} = 3\mathbf{i} + \mathbf{j}$  is
  - 5
  - 4
  - 12
  - 0.

**Paper Code: III-B**

Full Marks-80

Time: Three Hours Thirty Minutes

**Group-A****(Marks 20)**Answer any four questions.

5X4=20

1. Show that the vector  $(\vec{\beta} - \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}|^2} \vec{\alpha})$  is perpendicular to  $\vec{\alpha}$ .
2. If  $\hat{e}_1$  and  $\hat{e}_2$  be two unit vectors and  $\theta$  be the angle between them, then show that  $2\sin \frac{\theta}{2} = |\hat{e}_1 - \hat{e}_2|$ .
3. If  $|2\vec{a} + 3\vec{b}| = |2\vec{a} - 3\vec{b}|$ , then prove that the vectors  $\vec{a}$  and  $\vec{b}$  are orthogonal.
4. If  $\vec{a}, \vec{b}, \vec{c}$  represents three vectors determined by the sides of a triangle ABC taken in order, prove that  $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$ . Hence show that  $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$ .
5. If  $\vec{a} \times \vec{\beta} + \vec{\beta} \times \vec{\gamma} + \vec{\gamma} \times \vec{a} = \vec{0}$ , then show that  $\vec{a}, \vec{\beta}, \vec{\gamma}$  are coplanar.
6. Show that  $|\vec{a} \times \vec{\beta}|^2 |\vec{a} \times \vec{\gamma}|^2 - \{(\vec{a} \times \vec{\beta}) \cdot (\vec{a} \times \vec{\gamma})\}^2 = |\vec{a}|^2 [\vec{\alpha} \vec{\beta} \vec{\gamma}]^2$ .

**Group-B****(Marks 30)**Answer any four questions.

5X5=25

7. Show that if  $M(x, y)dx + N(x, y)dy = 0$ , is both homogeneous and exact, and  $Mx + Ny$  is not a constant, then its general solution is given by  $Mx + Ny = c$ .
8. If  $y_1$  &  $y_2$  be solutions of the differential equation  $\frac{dy}{dx} + Py = Q$ , where P & Q are functions of x alone and  $y_2 = y_1 z$ , then  $z = 1 + ae^{-\int y_1 Q dx}$ , 'a' being an arbitrary constant. If you think this is true write 1 otherwise write 0.
9. Solve  $y'' - y = e^x$  by undetermined coefficients and by variation of parameters.
10. Using the variation of parameter method solve  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$ .

11. Show that the system of co-axial parabolas  $y^2 = 4a(x + a)$  is self orthogonal.
12. Find the eigen values and eigen functions of  $\frac{d}{dx} \left( x \cdot \frac{dy}{dx} \right) + y \cdot \frac{\beta}{x} = 0$ , ( $\beta > 0$ ) satisfying the boundary conditions  $y(1) = 0$ ,  $y(e^\pi) = 0$ .
13. Find the complete integral of the equation by Charpit's method
- $$p^2x + q^2y = z, \text{ where } p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}.$$
14. Solve:  $\frac{dx}{dt} + 2x - 3y = t$ ,  $\frac{dy}{dt} - 3x + 2y = e^{2t}$ .
15. Solve the differential equation  $y = x - 2ap + ap^2$ . Find the singular solution and interpret it geometrically.

### Group-C

(Marks 30)

Answer any five questions.

7X5=35

16. Solve graphically the linear programming problem : Maximize  $z = 3x_1 + 4x_2$   
Subject to  $4x_1 + 2x_2 \leq 80$  ;  $2x_1 + 5x_2 \leq 180$  ;  $x_1, x_2 > 0$ .
17. Obtain the dual problem of the following L.P.P.  
**Maximize :**  $z = x_1 - 2x_2 + 3x_3$   
Subject to :  $-2x_1 + x_2 + 3x_3 = 2$   
 $2x_1 + 3x_2 + 4x_3 = 1$   
 $x_1, x_2, x_3 \geq 0$ .
18. Use simplex method to solve the following L.P.P.  
Maximize :  $y = 7x_1 + 5x_2$   
Subject to  $x_1 + 2x_2 \leq 6$  ;  $4x_1 + 3x_2 \leq 12$  ;  $x_1, x_2 \geq 0$ .
19. Solve the following Transportation problem

	A	B	C	Available
I	50	30	220	1
II	90	45	170	3
III	250	200	150	4
Requirement	4	2	2	

20. Solve the following assignment problem

	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

21. Solve the following  $5 \times 2$  game graphically :

		Player B	
		B <sub>1</sub>	B <sub>2</sub>
Player A	A <sub>1</sub>	-2	5
	A <sub>2</sub>	-5	3
	A <sub>3</sub>	0	-2
	A <sub>4</sub>	-3	0
	A <sub>5</sub>	1	-4