CHANCHAL COLLEGE

ASSIGNMENT- 2021

MATHEMATICS (HONOURS)

Paper Code: III-A

Full Marks-20 Time: Thirty minuits

Answer all the questions. Each question carries 2 marks.

1. Two circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2bx+c^2=0$ will touch each other if:

(A) $a+b=c$ (B) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ (C) $a^2+b^2=c^2$ (D) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

2. If PSQ is a focal chord of a parabola $y^2=8x$ and SP=6 unit then length of SQ is

(A) 3 unit (B) 4 unit (C) 6 unit (D) None of these

3. If the plane $x+y+z=c$ touches the sphere $x^2+y^2+z^2-2x-2y-2z-6=0$ then the value of C is (A) $3(1\pm\sqrt{3})$ (B) $3(2\pm\sqrt{3})$ (C) $\frac{(1\pm\sqrt{3})}{3}$ (D) $\frac{(2\pm\sqrt{3})}{3}$

4. The law of force towards the pole under which curves are described by $au=n\theta$ is

(A) $\frac{1}{r^3}$ (B) $\frac{1}{r^2}$ (C) $\frac{1}{r}$ (D) None of these

5. If the tangential and normal acceleration of a particle moving in a plane curve are equal, then expression of velocity is

(A) $v=Ae^{\theta}$ (B) $v=Ae^{-\theta}$ (C) $v=Ae^{2\theta}$ (D) None of these

6. The distance between the two planes $4x-8y+z=9$ and $4x-8y+z+18=0$ is

(A) 3 unit (B) 9 unit (C) 27 unit (D) None of these

7. Calculate the angular speed of flywheel making 420 revolutions per minute.

(A) 42300 rad/sec (B) 1200 rad/sec (C) 10/4200 rad/sec (D) 44 rad/sec

8. An aeroplane flying horizontally with a speed of 360km/h releases a bomb at a height of 490m from the ground. If g=9.8m/s^2, it will strike the ground at

(A) 10km (B) 100km (C) 1km (D) 16km

9. A line makes angle α, β, γ with x-axis, y-axis and z-axis respectively then **cos 2α + cos 2β + cos 2γ** is equal to

(A) 2 (B) 1 (C) -1 (D) NONE

10 . The direction ratios of the line of intersection of the planes 3x + 2y – z = 5 and x – y + 2z = 3 are

(A) 3, 2, -1 (B) -3, 7, 5 (C) 1, -1, 2 (D) – 11, 4, -5

Paper Code: III-B

Full Marks-80 Time: Three Hours Thirty Minutes

Group-A

(Marks 20)

Answer any four questions. 5X4=20

1. Prove that if one line of the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ be perpen-dicular to one line of the pair of straight lines represented by $a'x^2 + 2h'xy + b'y^2 = 0$ then $(aa' - bb') + 4(ah' + hb')(ha' + bh') = 0$.

2. Find the equation to the two circles that cut orthogonally the circles $x^2 + y^2 + 2x - 9 = 0$; and $x^2 + y^2 + 2x - 8x - 9 = 0$ and also touch the line . Hence find the distance between their centres.

3. Show that in a conic the semi-latus rectum is the harmonic mean between the segments of a focal chord.

4. If the tangents are drawn from any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the circle $x^2 + y^2 = r^2$, show that the chord of contact are tangent to the ellipse $x^2a^2 + y^2b^2 = r^4$.

5. Reduce the equation to normal form and determine the type of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$.

6. Show that the equation of the circle which passes through the focus of the parabola $\frac{2a}{r} = 1 + cos\theta$, and touches it at the point $\theta = \alpha$ is given by $rcos^3\frac{\pi}{2} = a\cos\left(\theta - \frac{3\alpha}{2}\right)$.

Group-B

(Marks 25)

Answer any four questions. 5X5=25

7. Show that the straight lines whose D.C's are given by the equations $al+bm+cn=0$ and $pl^2+qm^2+rn^2=0$ are perpendicular if $a^2(q+r)+b^2(r+p)+c^2(p+q)=0$ and are parallel if $\frac{a^2}{p}+\frac{b^2}{q}+\frac{c^2}{r}=0$.

8. A moving plane passes through a fixed point α,β,γ are intersects the coordinates axes at A, B, C show that the locus of the centroid of the Δ ABC is $\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=3$.

9. Find the equations of the image of the line $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-4}{2}$ in the plane $2x-y+z+3=0$.

10. Show that the lines $\frac{x-4}{5}=\frac{y-3}{-2}=\frac{z-2}{-6}$ and $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z-1}{-7}$ are coplanar. Find the point of intersection and the equation of the plane in which they lie.

11. Find the shortest distance and the equation of the line of shortest distance in symmetric form of the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

13. If the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes in A,B,C. Prove that the equation to the cone generated by lines drawn from 0 to meet the circle ABC is $\left(\frac{b}{c}+\frac{c}{b}\right)yz+\left(\frac{c}{a}+\frac{a}{c}\right)zx+\left(\frac{a}{b}+\frac{b}{a}\right)xy=0.$

14. Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as a great circle. Also find its centre and radius.

Group-C

(Marks 35)

Answer any five questions. 7X5=35

15. Show that the greatest height which a particle with initial velocity v can reach on a vertical wall at a distance 'a' from the point of projection is $\frac{v^2}{2g}-\frac{ga^2}{2v^2}$.

16. Obtain the equation to the central orbit in the form $\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2}$.

17. A short of mass m penetrates a thickness t of a fixed plate of mass M. If M were free to move and the resistance supposed to be uniform, show that the thickness penetrated is $\frac{Mt}{M+m}$.

18. Find the loss of kinetic energy due to direct impact of two spheres of masses m_1 and m_2 moving along a straight line with velocities u_1 and u_2.

19. The equation of a simple harmonic motion is $\frac{d^2x}{dt^2}=-\mu x$. Find x in terms of t.

20. Find the pedal equation to the following curves :

(i) Parabola whose pole at the focus, (ii) Ellipse whose pole at the focus.