CHANCHAL COLLEGE

ASSIGNMENT - 2021
MATHEMATICS (**GENERAL**)
Paper: MATH-G-DC-02

Full Marks : 32

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Notations and symbols have their usual meanings.

Group – A

Answer any four questions 1X4=4

1. (a) Solve (D2 – 4D + 4) y = 0.

(b) State D. AIembert's Ratio test for infinite Series of positive terms.

(c) State Lagrange's mean value theorem

(d) The series $\sum_{n=1}^{\infty} \frac{2}{e^n}$ is

(a)Convergent (b) divergent (c) oscillatory (d) none of these

(e) In MVT $f(h) = f(0) + hf'(\theta h)$ *in* $0 < \theta < 1$, *if* $f(x) = \frac{1}{1+x}$ *and* $h = 3$, *then find* θ?

Group-B

Answer any two questions 2X5=10

2. Solve ($x^2 D^2 + 4xD + 2$) y = x log x

3. Test the convergence of the series $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^2}{2^2} - \frac{3}{2}\right)^{-2} + \left(\frac{4^2}{3^2} - \frac{4}{3}\right)^{-3} + \ldots\ldots \infty.$

4. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, show that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

5. *Solve*: $(x^3 D^2 + 3x^2 D + 5x)\, y = 2.$

Group-B

Answer any two questions 2X9=18

6. If $I_n = \int x^n e^{-x} dx$, n being positive integer, show that $I_n = -x^n e^{-x} + nI_{n-2}$

Prove that , $\int_0^{\frac{\pi}{2}} log(sinx)\, dx = \frac{\pi}{2} \log(\frac{1}{2})$.

7. Solve by method of variation of parameter $y'' + 4y = \tan 2x$

8. Express $\int_0^1 x^p (1 - x^q)^m \, dx$ in terms of Gamma function. Hence Evaluate $\int_0^1 x^3 (1 - \sqrt{x}) dx$.

9. If $V_n = \frac{d^n}{dx^n}(x^n \log x)$ *show that* $V_n = nV_{n-1} + (n-1)!$ and hence show that

$$V_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n} \right)$$