CHANCHAL COLLEGE

ASSIGNMENT - 2021

MATHEMATICS (Honours)

Paper: MTMH - DC-04

Full Marks : 32 2^{nd} SEM

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Notations and symbols have their usual meanings.

Group – A

Answer any four questions 1X4=4

1. (a) Define the Coset of a group G.

(b)Write down the number of elements in S_n and A_6.

(c) Let G be a finite group of 84 elements. Find the size of a largest possible proper subgroup of G.

(d) What type of permutation (3 4 5 6), even or odd?

(e) Define Prime and maximal ideals.

Group-B

Answer any two questions 2X5=10

2. State And prove first isomorphism theorem.

3. Show that for Rings If $f: R \to S$ is a ring homomorphism with kernel K, then the image of **f** is isomorphic to **R/K**.

4. Show that every proper ideal I of the ring R is contained in a maximal ideal. Consequently, every ring has at least one maximal ideal.

5. Prove that if *f and g* are polynomials in R[X], with *g* monic, there are unique polynomials q and r in R[X] such that $f = q\,g + r$ *and* $\deg r < \deg g$. If R is a field, g can be any nonzero polynomial.

Group-C

Answer any two questions 2X9=18

6.a) Let G be the additive group of integers. Then prove that the set of integers of multiple of 6 is a subgroup of G.

b) If ***a*** is prime, then ***a*** is irreducible, but not conversely test the validity of the statement.

7. Let H and N be subgroups of G, with N normal in G. Then prove that

   (i) HN = NH, and therefore, HN is a subgroup of G.

   (ii) N is a normal subgroup of HN.

   (iii) $H \cap N$ is a normal subgroup of H.

8. a) Find all homomorphisms from $(\mathbb{Z}_8, +)$ *into* $(\mathbb{Z}_6, +)$

   b) prove that $\mathbb{Z}_9$ is not a homiomorphic image of $\mathbb{Z}_{16}$.

9.a) Let G, G_1 be two groups. $f: G \to G_1$ be an isomorphism. Prove that G is commutative iff G_1 is commutative.

b) Define a normal subgroup of a group, Let G be a group and H be a subgroup of G. If for all $a, b \in G$, $ab \in H$ implies $ba \in H$. Prove that H is a normal subgroup of G.