U.G. 3rd Semester Examination 2021

MATHEMATICS (Honours)

Paper : DC-6

[Linear Algebra]

(CBCS)

Full Marks : 32 Time : 2 Hours

The figures in the margin indicate full marks.
Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any ***four*** questions : $4\times1=4$

(a) If A be a 3×3 matrix with α and β be only two eigen values ($\alpha \neq \beta$), then find the characteristic polynomial of A.

(b) Examine if the set $S = \{(x, y, z) \in R^3 : x + y + z = 1\}$ is a subspace of R^3.

(c) For what real values of K does the set $S = \{(K, 1, 1), (1, K, 1), (1, 1, K)\}$ form a basis of R^3.

(d) Find the matrix representation of a linear mapping $T : R^2 \to R^2$ defined by $T(x, y) = (x + y, x - y)$ relative to the basis $\{(1, 0), (0, 1)\}$ of R^2.

(e) The matrix $A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$ has an eigen vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find the corresponding eigen value.

(f) If α, β be any two vectors in a Euclidean space V, then prove that, $\|\alpha+\beta\|^2+\|\alpha-\beta\|^2=2\|\alpha\|^2+2\|\beta\|^2$.

(g) Let $T : R^2 \to R^3$ be a linear transformation given by $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$. Then find the Rank T.

Group - B

(10 Marks)

Answer any ***two*** questions : 2×5=10

2. If U and W be two subspaces of a vector space V over a field F, then the union $U \cup W$ is a subspace of V iff either $U \subset W$ or $W \subset U$. 5

3. Apply Gram Schmidt Process to the set $\{(1, 1, 1), (2, -2, 1), (3, 1, 2)\}$ to obtain an orthonormal basis of R^3 with standard inner product. 5

4. Prove that the mapping $T : R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (-2x_1 + x_2, -x_1 + 2x_2 + 4x_3, 3x_1 + x_3)$ is a linear transformation. Find the matrix of T in the ordered basis $(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = (-1, 2, 1)$, $\alpha_2 = (2, 1, 1)$ and $\alpha_3 = (1, 0, 1)$. 5

5. If $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$, then use Cayley-Hamilton theorem to express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A.

Group - C

(18 Marks)

Answer any ***two*** questions : 2×9=18

6. (a) Find an orthogonal matrix which diagonalises $A = \begin{pmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{pmatrix}$. 5

   (b) Prove that the eigen vectors corresponding to two distinct eigen values of a real symmetric matrix are orthogonal. 4

7. (a) Show that the mapping $T : R^3 \to R^3$ defined by $T(x, y, z) = (x - y + 2z, x - 2y + 2z, 2x + y)$ is an isomorphism. 4

   (b) Let V and W be vector spaces of equal (finite) dimension and $T : V \to W$ be linear. Prove that T is one-to-one if and only if Rank (T) = dim (V).

8. (a) Prove that in a Euclidean space V, two vectors α, β are linearly dependent if and only if $|(\alpha, \beta)| = \|\alpha\| \, \|\beta\|$. 4

(b) Find a basis and determine the dimension of the subspace, $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in R_{2\times2} : a + b = 0 \right\}$ of the vector space $R_{2\times2}$. 5