U.G. 4th Semester Examinations 2022

MATHEMATICS (Honours)

Paper Code : DC-08

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

[DIFFERENTIAL EQUATIONS]

Group-A

1. Answer any ***four*** questions : 1×4=4

(a) Show that $\frac{1}{3x^3y^3}$ is an integrating factor of $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$.

(b) Find integrating factor of the differential equation $\left(x^2y-2xy^2\right)dx-\left(x^3-3x^2y\right)dy=0$

(c) Solve : $(x+3y+2)\frac{dy}{dx}=1$

(d) Solve : $y=px-\sin^y p$, $p=\frac{dy}{dx}$

(e) Find particular integral of $\left(D^2+D-2\right)y=e^x$, $D\equiv\frac{d}{dx}$

(f) Is e^x an integral of the homogeneous equation $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0$?

(g) Obtain partial differential equation from $z=f(\sin x+\cos y)$.

Group-B

Answer any ***two*** questions : 5×2=10

2. Solve : $\left(x^2y-2xy^2\right)dx-\left(x^3-3x^2y\right)dy=0$

3. Find the power series solution of $\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+3y=0$ about x = 0.

4. Solve : $\left(D^2-2D+3\right)y=\sin x$, using method of undetermined coefficients.

5. Solve : $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$, given that $y(0)=0,\ y'(0)=2$.

Group-C

Answer any ***two*** questions : 9×2=18

6. (a) Solve : $(x+1)\frac{d^2y}{dx^2}-2(x+3)\frac{dy}{dx}+(x+5)y=e^x$

(b) Use Charpit's method to find the complete and singular integrals of the PDE $\left(p^2+q^2\right)y=qz$. 5+4

7. (a) Solve : $\frac{d^2y}{dx^2}+a^2y=\tan ax$ by the method of variation of parameter.

(b) Find eigen values and eigen functions of the differential equation $\frac{d^2y}{dx^2}+\lambda y=0.\ (\lambda>0)$ satisfying the boundary conditions $y(0)=y(1)$ and $y'(0)=y'(1)$. 4+5

8. (a) Solve : $(mz-ny)p+(nx-lz)q-ly-mx$, by Lagrange Method,

where $p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y}$.

(b) Show that $P_{2n}(O)=(-1)^n\frac{(2n)!}{2^{2n}(n!)^2}$, where $P_n(x)$ denotes Legendre's polynomial.

5+4