UG/6th Sem (H)/22 (CBCS)

U. G.6th Semester Examinations 2022

MATHEMATICS (Honours)

Paper Code : DSE - 3A/3B/3C

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

DSE-3A

[POINT SET TOPOLOGY]

Group-A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

(a) Give an example to show that union of two topologies on a nonempty set may not be a topology.

(b) Which sets in a discrete topological space are closed?

(c) If X be a finite set and τ_1, τ_2 be discrete topology and cofinite topology respectively. Compare τ_1 and τ_2.

(d) Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, x, \{b\}, \{b, c\}, \{b, c, d\}\}$ be a topology on X. Examine the connectedness of X.

(e) State continuum hypothesis.

(f) Give example of a compact subset in $\mathbb{R}$ with usual topology.

(g) Find a basis for discrete topology on a set.

Group-B

(10 Marks)

Answer any ***two*** questions : 5×2=10

2. Let (X, τ) be a topological space. $\phi \neq Y \subseteq X$. Show that $\tau_y = \{U \cap Y : U \in \tau\}$ forms a topology on Y. 5

3. If (X, τ) is a topological space and *A, B* are any two subsets of *X,* then show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. 5

4. Let (X, τ_1) and (Y, τ_2) be two topological space and $f : X \to Y$ be a continuous mapping. Then show that *f* carries compact set of (X, τ_1) to a compact set of (Y, τ_2).

5. Let (X, τ_x) and (Y, τ_y) be two topological spaces. Show that $f : X \to Y$ is continuous if and only if for every closed subset $V \subseteq Y$, the set $f^{-1}(V)$ is closed in *X.* 5

Group-C

(18 Marks)

Answer any ***two*** questions : 9×2=18

6. (a) Let (X, τ) be a topological space. $Y \subseteq X$, (Y, τ_y) be subspace of (X, τ). If *F* be a closed set in (X, τ) then show that $F \cap Y$ is closed set in (Y, τ_y) and conversely. 4

   (b) Prove that a subfamily β of a topology τ on a set *X* be a base for τ iff each number of τ be the union of members of β. 5

7. (a) If $X_1, X_2, \ldots\ldots, X_n$ are topological spaces and $\beta_1, \beta_2, \ldots\ldots, \beta_n$ are bases respectively, then prove that $\beta = \{u_1 \times u_2 \times \ldots\ldots \times u_n : u_1 \in \beta_1, u_2 \in \beta_2, \ldots. u_n \in \beta_n\}$ is a base of $X = X_1 \times X_2 \times \ldots . \times X_n$. 5

   (b) State and prove Hausdorffs Maximal principle. 4

8. (a) In a topological space (X, τ), show that closure of a set is the intersection of all the closed sets containing the set. 4

   (b) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$. Find int $(\{b, c\}), \{\overline{a}\}$ and $\{\overline{b, c}\}$. 2+2+1

DSE - 3B

[CBCS]

[THEORY OF ORDINARY DIFFERENTIAL EQUATION]

Group-A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

(a) Sketch phase portraits of stable and unstable node.

(b) Discuss the existence and uniqueness of solutions for the IVP $ty^1 = t + |y|, y^{(-1)} = 1$.

(c) Express the differential equation $\frac{d^4y}{dt^4} - y = 0$ in the form $\dot{\vec{x}} = A\vec{x}$.

(d) Find the maximal interval of existence of the equation $\dot{x} = x^2$ with $x(0) = 1$.

(e) If A be a square matrix, then prove that $\frac{d}{dt}e^{At} = Ae^{At}$.

(f) Find the Jordan canonical form of the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

(g) Investigate the stationary point $x = 0$, $y = 0$ of the system

$$\dot{x} = 2x + y - 5y^2$$
$$\dot{y} = 3x + y + \frac{x^3}{2}$$

for stability in first approximation.

Group-B

(10 Marks)

Answer any ***two*** questions : 5×2=10

2. (a) Using Lyapunov function investigate the stability of the trivial solution of the system

$$\frac{dx}{dt} = -x - 2y + x^2y^2$$

$$\frac{dy}{dt} = x - \frac{y}{2} - \frac{x^3y}{2}$$

(b) State Lyapunov's stability theorem. 3+2

3. Find the general solution and draw the phase portrait of the linear system

$$\dot{x}_1 = x_1$$
$$\dot{x}_2 = 2x_2$$

4. If $\phi(t)$ be the fundamental matrix solution of the T-periodic system $\dot{x} = Ax$ then there exist a non-singular constant matrix B such that

Let $B = \exp\left[\int_0^T tr.(A(s))ds\right]$

5. Find the first four successive approximations $u^{(1)}(t,a)$, $u^{(2)}(t,a)$, $u^{(3)}(t,a)$ and $u^{(4)}(t,a)$ for the system

$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_2 + x_1^2$$
$$\dot{x}_3 = x_3 + x_2^2$$

Show that $u^{(3)}(t,a) = u^{(4)}(t,a) = \ldots\ldots$ and hence $u(t,a) = u^{(3)}(t,a)$. Find the stable and unstable manifolds S and U for this problem. 5

Group-C

(18 Marks)

Answer any ***two*** questions : 9×2=18

6. (a) Prove that the regular system $\dot{x} = P(t)x$ where P is an $n \times n$ matrix function with minimal period T, has atleast one non-trivial solution $x = \psi(t)$ such that $\psi(t+T) = \mu\psi(t)$, $-\infty < t < \infty$. Where μ is a constant.

   (b) Prove also that the constant μ is independent of the choice of Φ. 6+3

7. State and prove the fundamental existence uniqueness theorem. 9

8. Using Liapunov function show that the origin is an asymptotically stable equilibrium point of the system.

$$\dot{\underset{\sim}{x}} = \begin{bmatrix} -x_2 - x_1x_2^2 + x_3^2 - x_1^3 \\ x_1 + x_3^3 - x_2^3 \\ -x_1x_3 - x_3x_1^2 - x_2x_3^2 - x_3^5 \end{bmatrix}$$

Show that the trajectories of the linearized system $\dot{\underset{\sim}{x}} = D\underset{\sim}{f}(0)\underset{\sim}{x}$ for this problem lie on the circles in planes parallel to the x_1, x_2 plane; hence, the origin is stable, but not asymptotically stable for the linearized system. 5+4

DSE - 3C

[CBCS]

[INTEGRAL TRANSFORM]

Group-A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

(a) State and prove the second translation theorem for Laplace transform.

(b) Evaluate Fourier sine transform of $f(x)=\frac{1}{x}$.

(c) If $F(f(x))=\overline{f}(p)$, then find $F\{f(ax)\}=?$

(d) Show that if $f_c(s)$ is the Fourier cosine transform of $F(x)$, then show that Fourier cosine transform of $F\left(\frac{x}{a}\right)$ is $af_c(as)$.

(e) Write down the left shift theorem for z-transform.

(f) Find $L^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\}$.

(g) Use linearity property of Z-transformation to find $Z\{\sinh n\}$.

Group-B

(10 Marks)

Answer any ***two*** questions : 5×2=10

2. Evaluate $\int_0^\infty te^{-3t}\cos(4t)\,dt$, using Laplace transformation.

3. Establish the relation between Fourier transform and Laplace transform.

4. Find the Fourier sine and cosine transform of $\frac{e^{ax}+e^{-ax}}{e^{\pi x}-e^{-\pi x}}$.

5. Let the sequence $\{f_n\}$ be defined as $f_n - \frac{e^{-n}}{n!}$. Find the Z-transform of f_n i.e $Z\{f_n\}$.

Group-C

(18 Marks)

Answer any ***two*** questions : 9×2=18

6. (a) Find the cosine transform of a function of x which is unity for $0 < x < a$ and zero for $x \geq a$. What is the function whose cosine transform is $\frac{\sin as}{s}\left(or\ \frac{\sin ap}{p}\right)$? 5

(b) Solve the integral equation $\int_0^\infty F(x)\cos(sx)\,dx = \begin{cases} 1-s, & 0 \leq s \leq 1 \\ 0, & s > 1 \end{cases}$.

Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \frac{\pi}{2}$. 4

7. (a) Apply Laplace transform to solve $\frac{d^2y}{dt^2} + y = 6\cos 2t$ gives that $y = 3,\ \frac{dy}{dt} = 1$ when t = 0. 5

(b) Use convolution theorem to prove that

$$L^{-1}\left\{\frac{16}{p\left(p^2+4\right)^2}\right\} = \int_0^t (\sin 2\alpha - 2\alpha\cos 2\alpha)\,d\alpha.$$ 4

8. (a) Solve the difference equations using z transforms of the following

$y_{n+2} - 3y_{n+1} + 2y_n = 0,\ y_0 = -1, y_1 = 2$. 5

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$ 4
