

FYUGP/1st Sem/23(NEP)

2023

Four Year Under Graduate Programme (FYUGP)

1st Semester Examination (Under NEP)

**MATHEMATICS (Major)**

**Paper Code : MTMMJ MC-01**

**(Calculus & Geometry)**

Full Marks : 40

Time : Two Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

**Group - A**

(Marks - 10)

1. Answer any five questions : 2×5=10

(a) Use Leibnitz's formula to find the  $n$ th order derivative of  $x^3 e^{ax}$ .

(b) Show that  $\lim_{x \rightarrow 0} \frac{1}{x} \sin\left(\frac{1}{x}\right)$  does not exist.

(c) A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by

$$f(x) = \begin{cases} \cos \frac{1}{x}, & x \neq 0, \\ 0, & x = 0 \end{cases}$$

Prove that  $f$  is not continuous at 0.

P.T.O.

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- (d) Find the length of the curve  $y = \frac{a}{2}(e^{x/a} + e^{-x/a})$  between the points  $x = 0$  and  $x = x$ .
- (e) Find the centre and the radius of the sphere  $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$ .
- (f) Find the equation of the normal at the point  $(1, 1)$  to the hyperbola  $x^2 - 2y^2 + x + y = 1$ .
- (g) Find the midpoint of the chord of  $y^2 = 4x$  which lie on the straight line  $y = 5x - 1$ .

**Group - B**

(Marks - 30)

Answer any six questions :

5×6=30

2. If  $y = e^{ax} \cos^2 bx$ , find  $y_n$ .

3. Prove that

$$B(x, y) = \int_0^1 \frac{t^{x-1} + t^{y-1}}{(1+t)^{x+y}} dt = \frac{1}{2} \int_0^\infty \frac{t^{x-1} + t^{y-1}}{(1+t)^{x+y}} dt$$

4. If  $I_n = \int_0^1 x^n \tan^{-1} x dx$ , then show that

$$(n+1)I_n = \frac{\pi}{2} - \frac{1}{n} - (n-2)I_{n-2}$$

5. Find the equation of the cubic which has the same asymptotes as the curve  $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$  and which passes through  $(0, 0)$ ,  $(1, 0)$  and  $(0, 1)$ .
6. Show that the surface area of the solid generated by revolving the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$  about the line  $y = 0$ , is  $\frac{64}{3}\pi a^2$ .
7. Show that the plane  $y + 3 = 0$  intersects the hyperbolic paraboloid  $\frac{x^2}{5} - \frac{y^2}{4} = 6z$  in a parabola. Find the vertex and the length of the latus rectum of the parabola.
8. Prove that the two conics  $\frac{l_1}{r} = 1 - e_1 \cos \theta$  and  $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$  will touch one another, if  $l_1^2(1 - e_2^2) + l_2^2(1 - e_1^2) = 2l_1l_2(1 - e_1e_2 \cos \alpha)$ .
9. Show that the circles  $x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$ ,  $5y + 6z + 1 = 0$  and  $x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0$ ,  $x + 2y - 7z = 0$  lie on the same sphere, whose equation is  $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$ .

10. Reduce the equation  $3x^2 - 24y^2 + 8z^2 + 16yz - 10zx - 14xy + 22y + 2z - 4 = 0$  to the canonical form and find its axis.

বঙ্গানুবাদ

বিভাগ - ক

(পূর্ণমান - ১০)

1. যে কোনো পাঁচটি প্রশ্নের উত্তর দাও : ২×৫=১০

(a) Leibnitz সূত্র প্রয়োগ করে  $x^3 e^{ax}$ -এর  $n$ -তম ক্রমের অবকল সহগ নির্ণয় করো।

(b) দেখাও যে,  $\lim_{x \rightarrow 0} \frac{1}{x} \sin\left(\frac{1}{x}\right)$ -এর অস্তিত্ব নেই।

(c) একটি অপেক্ষক  $f : \mathbb{R} \rightarrow \mathbb{R}$  নিম্নলিখিত রূপে সংজ্ঞাত :

$$f(x) = \cos \frac{1}{x}, \quad x \neq 0,$$

$$= 0, \quad x = 0$$

প্রমাণ করো যে,  $f$ ,  $x = 0$  বিন্দুতে সম্মত নয়।

(d)  $x = 0$  এবং  $x = x$  বিন্দু দুটির মধ্যে  $y = \frac{a}{2}(e^{x/a} + e^{-x/a})$  বক্ররেখার দৈর্ঘ্য নির্ণয় করো।

(e)  $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$  গোলকটির কেন্দ্রবিন্দু ও ব্যাসার্ধ নির্ণয় করো।