28th July, 2023
(1 pm - 3 pm)

UG/6th Sem(H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MATH6 - SEC-2

Full Marks : 32 | Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *four* questions. $1\times4=4$

1. (a) Let X be a Poisson variate with parameter μ and $P(X=0)=P(X=1)$, prove that $\mu=1$. 1

(b) Find the mean of the random variable X whose density function $f(x)$ is given by 1

$$f(x)\begin{cases} e^{-x}\,;\ 0<x<\infty \\ 0\ ;\text{elsewhere}\end{cases}$$

(c) If the lines of regression of y on x and x on y are $3x+2y=26$ and $6x+y=31$ respectively. Find the correlation coefficient between x and y. 1

(d) A die is thrown 108 times in succession. Find the expectation of the number of 'six' appeared. 1

P.T.O.

(e) Find the probability that there may be 53 Sundays in a leap-year. 1

(f) The coefficient of variation is 40 and the mean is 30; find the standard deviation. 1

(g) Define scatter diagram. 1

Group - B

Answer any *two* questions. 5×2=10

2. There are two identical boxes. The first box contains 5 white, 7 red balls and the second box contains 5 white, 5 red balls. One box is chosen at random and a ball is drawn from it. If the ball drawn is found to be white, calculate the probability that it is drawn from the first box. 5

3. Calculate the mean deviation from the mean of the following distribution —

Marks	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

5

4. The scores of two batsmen, A and B, in ten innings during a certain season, are as under —

A:	32	28	47	63	71	39	10	60	96	14
B:	19	31	48	53	67	90	10	62	40	80

Find which of the batsman is more consistent in scoring.

5. Let X be a Poisson variate with parameter μ. Show that $P(X \le n) = \frac{1}{n!}\int_0^\alpha e^{-x} x^n dx$, where n is any positive integer. 5

Group - C

Answer any *two* questions : 9×2=18

6. (a) A coin is tossed $(m + n)$ times $(m > n)$. Show that the probability of getting at least m consecutive heads is $\frac{n+2}{2^n+1}$. 5

(b) The I.Q. of students of a class is normally distributed with parameter $m = 100$ and $\sigma = 10$. If the total number of students in the class is 700, then find the number of students who have $I.Q. \ge 115$. Given that $\frac{1}{2\Pi}\int_{-\alpha}^{1.5} e^{\frac{-x^2}{2}} .dx = 0.9332$. 4

7. (a) Find out the skewness and Kurtosis of the series by the method of moments : 5

Measurement	0-10	10-20	20-30	30-40
Frequency	1	3	4	2

P.T.O.

(b) Using the method of least square, fit a curve of the $y = a + bx^2$ to the following data — 4

x	0	1	2	3
y	1	6	20	48

8. (a) For the Binomial (n, p) distribution, prove that

$$\mu_{r+1} = p(p-1)\left[nr\,\mu_{r-1} + \frac{d\mu r}{dp}\right]$$ 5

where μ_r is the rth central moment of the distribution.

(b) If the random variables X and Y are connected by the linear relation $2x+3y+4=0$. Show that $\rho(x,y) = -1$. 4

(24th July, 2023) (1 pm - 3 pm)

UG/6th Sem(H)/23/(CBCS)

2023

MATHEMATICS (Honours)

Paper Code : MATH6 - DC-13

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any *four* questions : $1\times4=4$

   (a) When a feasible solution of an L.P.P called an optimal solution?

   (b) State fundamental theorem of L.P.P.

   (c) How many basic solutions are possible in a system of m-equations and n-unknowns? $(n \geq m)$

   (d) Define a convex set.

   (e) State why an assignment problem is not an LPP.

   (f) Find the extreme points, if any, of the following set :

$$S=\{(x,y):x^2+y^2\leq 25\}$$

   (g) Define saddle point.

P.T.O.

Group - B

Answer any *two* questions : 5×2=10

2. Show that intersection of two convex sets is also a convex set.

3. Solve the following LPP graphically

Maximize $Z = 4x + 2y$

Subject to $3x + y \geq 27$
$-x - y \leq -21$
$x + 2y \geq 30, \quad x, y \geq 0$

4. Prove that the dual of the dual of a given primal is primal.

5. Solve the following 2 × 4 game problem graphically :

		B			
		B_1	B_2	B_3	B_4
A	A_1	1	3	0	2
	A_2	3	0	1	−1

Group - C

Answer any *two* questions : 9×2=18

6. Find the solution of the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	10	20	5	7	10
O_2	13	9	12	8	20
O_3	4	15	7	9	30
O_4	14	7	1	0	40
O_5	3	12	5	19	50
b_j	60	60	20	10	

Examine whether the problem has an alternative optimal solution. 7+2

7. (a) Solve the following L.P.P by simplex method. 5

Maximize $Z = 7x_1 + 5x_2$

Subject to $x_1 + 2x_2 \leq 6$

$4x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0.$

(b) Solve the following assignment problem 4

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

P.T.O.

8. (a) Use dominance property to solve the following problem of game 7

		B			
		B_1	B_2	B_3	B_4
	A_1	4	2	3	2
A	A_2	–2	4	6	4
	A_3	2	1	3	5

(b) Explain the concepts of pure strategies. 2

Paper Code : MATH6 - DSE-3(A), 3(B) & 3(C)

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Paper Code : DSE-3(A)

(Point Set Topology)

Group - A

1. Answer any *four* questions : 1×4=4

(a) Let $\mathbb{R}_u$ and $\mathbb{R}_l$ denote respectively the usual topology and the lower limit topology on $\mathbb{R}$. Is the identify function $f:\mathbb{R}_u \to \mathbb{R}_l$ continuous?

(b) Write down a basis for the discrete topology on a non-empty set x.

(c) Find all the limit points of $\{b, c\}$ in the topological space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}\}$.

(d) Find if there exist any set which is neither open nor

P.T.O.

closed in the topological space (X, τ) where $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b,c\}\}$.

(e) State continuum hypothesis.

(f) Give example of a path connected topological space.

(g) State the Baire category theorem.

Group - B

Answer any *two* questions : 5×2=10

2. State and prove Schroeder-Bernstein theorem.

3. Let Y be a subspace of a topological space X and A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Show that the closure of A in Y is $\overline{A} \cap Y$.

4. [illegible] a collection o[illegible] subspace of a topologi[illegible]l space, that hav[illegible] common is connected.

(f) Give example of a path connected topological space.

(g) State the Baire category theorem.

Group - B

Answer any *two* questions : 5×2=10

2. State and prove Schroeder-Bernstein theorem.

3. Let Y be a subspace of a topological space X and A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Show that the closure of A in Y is $\overline{A} \cap Y$.

4. Prove that the union of a collection of connected subspace of a topological space, that have a point in common is connected.

5. Prove that closure of a set is the smallest closed set containing the set.

Group - C

Answer any *two* questions : 9×2=18

6. (a) State Zorn's lemma. Hence prove the Hausdroff Maximal principle. 1+4

(b) Let $\mathcal{B}$ be a basis for a topology τ on X, prove that τ equals to the collection of all unions of elements of $\mathcal{B}$. 4

7. (a) If $\mathcal{B}$ and $\mathcal{C}$ are basis of two topologies on X and Y respectively, then show that the collection $\mathcal{D}=\{B\times C : B\in\mathcal{B}, C\in\mathcal{C}\}$ is a basis for the topology on $X\times Y$. 5

(b) Prove that image of a compact set under a continuous mapping is compact. 4

8. (a) Let $X=\{1,2,3\}$ and $Y=\{a,b,c\}$. Let τ_1 and τ_2 be topologies on X and Y respectively, where $\tau_1=\{\phi, X, \{1\}\}$ and $\tau_2=\{\phi, Y, \{a\}, \{a,b\}\}$. A map f is defined by $1\to a, 2\to a, 3\to b$. Examine whether the mapping f is open, closed, continuous. 5

(b) State and prove the Ascoli-Arzela theorem. 4